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HW III , Math 530, Fall 2014

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QUESTION 1. (i) Let $(G, *)$ be a group of even order. Show that there exists $a \in G$ such that $a^2 = e$.

(ii) Let $(G, *)$ be a group and $a, b \in G$. Suppose that $a * b = b * a^{-1}$ and $b * a = a * b^{-1}$. Show that $a^4 = b^4 = e$.

(iii) Prove that a group $(G, *)$ is abelian if and only if $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$.

(iv) Let $(G, *)$ be a group and $a \in G$. Suppose that $a^n = e$ for some integer $n \geq 2$. Prove that $|a|$ must divide n.

(v) Let $(G, *)$ be a group and $a, b \in G$. Show that $(a * b * a^{-1})^n = a * b^n * a^{-1}$ for all integers n.

(vi) Let $A = \{1, 2, 3\}$, and let $P(A)$ be the power set of A. We know that $|P(A)| = 8$. Define a binary operation $*$ on $P(A)$ such that $x * y = (x - y) \cup (y - x)$ for every $x, y \in P(A)$. Prove that $(P(A)), *)$ is a group (Do not show associative, closure...these are clear). Construct a subgroup H of $(P(A), *)$ that has exactly 4 elements. Prove that $P(A)/H$ is a group by constructing the Caley-table of $P(A)/H$.

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